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Atomic and Gravitational Clocks

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Atomic and gravitational clocks are governed by the laws of electrodynamics and gravity, respectively. While the strong equivalence principle (SEP) assumes that the two clocks have been synchronous at all times, recent planetary data seem to suggest a possible violation of the SEP. Our past analysis of the implications of an SEP violation on different physical phenomena revealed no disagreement. However, these studies assumed that the two different clocks can be consistently constructed within the framework. The concept of scale invariance, and the physical meaning of different systems of units, are now reviewed and the construction of two clocks that do not remain synchronous—whose rates are related by a nonconstant function β_a—is demonstrated. The cosmological character of β_a is also discussed.

1. INTRODUCTION

Of the forces of nature, the two more successfully described are the electromagnetic forces, through quantum electrodynamics (QED), and macroscopic gravity, through Einstein's theory of general relativity (GR). The high level of agreement between predictions and observations leave little doubt that we now possess the correct physical interpretation as well as the theoretical tools to describe both forces.

QED and GR are also complete theories in the sense that they yield operationally well-defined clocks which satisfy the dynamical equations of the theories themselves. To understand what is required for a theory to yield a well-defined clock, we introduce the notion of scale invariance, SI.

Consider a dynamical equation defined in a given system of units, containing field variables and dimensional parameters. Consider now a scale (length) transformation of the type

$$L \to L_* = \Omega_*^{-1}(x) L \tag{1}$$

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where $\Omega_*(x)$ is a dimensionless arbitrary function of space-time. Regarding coordinates as dimensionless space-time markers, it follows from equation (1) that

$$g^*_{\mu\nu} = g_{\mu\nu}\Omega_*^{-2}(x), \qquad ds_* = \Omega_*^{-1}\, ds \tag{2}$$

Furthermore, a physical tensor Λ of arbitrary rank will be taken to transform like

$$\Lambda \to \Lambda^* = \Lambda\Omega_*^{-\pi(\Lambda)} \tag{3}$$

where $\pi(\Lambda)$ is called the power of Λ: $\pi(ds) = 1$, $\pi(g_{\mu\nu}) = 2$. Because null cones transform into null cones, it follows that $c = 1$ holds in all systems of units. Therefore, because $v = (v/c)c$, $\pi(v) = 0$. The power of any quantity can therefore be expressed in terms of $\pi(m)$ and $\pi(L) = 1$.

Given these definitions, an equation is scale invariant if: (1) in the transformed system of units, it preserves the same form involving the transformed fields; (2) it has the same parameters; and (3) there is no explicit dependence on $\Omega_*(x)$.

We now show that an SI theory cannot yield a unit of time, that is, a clock. Because of the assumed SI, the dynamical equations governing the clock are identical in any system of units and so their solution for the period p of the clock in one given system of units is the same in any other system of units, $p = p_*$. On the other hand, if instead of solving the dynamical equations, we apply the transformation (1) directly to p, we obtain the result $p = \Omega_* p_*$, that is $p \neq p_*$, in contradiction with the previous result. It follows that a SI theory yields a solution that is simultaneously constant and variable, thus proving that such a quantity is physically meaningless. In other words, an SI theory, being invariant with respect to changes in scale, does not possess a scale, thus lacking an intrinsic unit of time.

Therefore, an SI theory cannot provide a clock. For a clock to exist, the underlying theory must be non SI.

As an example, let us consider the case of the electromagnetic clock, an electron revolving around a proton, governed by the following equations (Weinberg, 1972):

$$\frac{1}{\sqrt{g}}[\sqrt{g}\, F^{\mu\nu}]_{,\nu} = J^\mu, \qquad u^\mu_{;\nu}u^\nu = \frac{e}{m}\, u^\lambda F^\mu_\lambda \tag{4}$$

The periodic solution with the period p is given by

$$p = 2\pi\frac{m^2 l^3}{e^4}\left[1 + 0\left(\frac{v}{c}\right)^2\right] \tag{5}$$

where l is the angular momentum per unit mass. In equations (4) there is a "time coordinate" x^0 whose physical meaning is not given by equations

(4), where x^0 is in fact a marker. Its physical meaning can be determined by considering that since $de/dx^0 = dm/dx^0 = dl/dx^0 = 0$, we have from equation (5) $dp/dx^0 = 0$, thus allowing one to attribute a physical meaning physical unit of time, given by equation (5). This property is due to the fact that equations (4) are not SI. In fact, let us apply equation (1) to equations (4). As $2\pi(e) = \pi(h) = 1 + \pi(m) = 2 - \pi(G) \equiv 2 - g$ and $2\pi(F_\alpha^\beta) = 2\pi(E, H) = \pi(\rho_\gamma) = \pi(m) - 3$ (E and H are the electric and magnetic field strengths, ρ_γ is the energy density, e^2/hc is a pure number, and both h/mc and GM/c^2 have the dimensions of a length), equations (4) become, treating coordinates as dimensionless,

$$\frac{1}{\sqrt{g_*}}[\sqrt{g_*}\, F_*^{\mu\nu}\Omega_*^{4+\pi}]_{,\nu} = J_*^\mu \Omega_*^{4+\pi} \tag{6a}$$

$$u_{;\nu}^{*\mu} u^{*\nu} + \Delta_*^{\mu\nu} \frac{\Omega_{,\nu}^*}{\Omega_*} = \frac{e}{m} u^{*\lambda} F_{*\lambda}^{\mu} \Omega_*^{\pi+3} \tag{6b}$$

with $\Delta_{\mu\nu} \equiv u_\mu u_\nu - g_{\mu\nu}$ and $2\pi \equiv 2\pi(F^{\mu\nu}) = \pi(m) - 7$.

Clearly, while equation (6a) can be made SI by assuming $\pi(m) = -1$, equation (6b) cannot, for the Ω_* dependence in the Δ term cannot be made to disappear, because there are no free parameters in $\Delta_{\mu\nu}$. Therefore the system (4) is not SI.

Let us now consider a gravitational clock (a planet and a star, for example) and let us consider the Einstein equations describing macroscopic gravity. From the previous analysis it follows that the lack of SI of equations (4) is due solely to the equations of motion, which in the gravitational case are already contained in the Einstein field equations, which can therefore be expected to be not SI. This is indeed the case (Singe, 1976): the Einstein equations do not retain their original form under the scale transformations (1)-(2). Their lack of SI is why a gravitational clock is a well-defined quantity, whose period, given by Kepler's third law, reads (M is the total mass and J is the orbital angular momentum per unit mass)

$$P = \frac{2\pi J^3}{(GM)^2} \tag{7}$$

The above discussion shows that the electromagnetic and gravitational clocks are meaningful because the dynamical equations governing them are not SI.

2. THE STRONG EQUIVALENCE PRINCIPLE

To study the relationship of the two clocks, consider a physical phenomenon characterized by a proper length interval Δs, which we shall measure using electromagnetic and gravitational clocks, the results being Δs_a (atomic) and Δs_E (Einstein), respectively.

Because clocks are the manifestation of underlying forces and we do not yet possess a unified theory, we do not know *a priori* whether the ratio

$$\frac{\Delta s_E}{\Delta s_a} = \beta_a \tag{8}$$

is constant. The lack of knowledge of the function β_a has so far been circumvented by adopting the strong equivalence principle (SEP) (Will, 1979; Thorne *et al.*, 1973), which assumes that

$$\beta_a = \text{const} = 1 \tag{9}$$

implying that, for example, the value of the period of a planet should be independent of the clock used to measure it.

The SEP comprises two assumption (Will, 1979): (1) that the weak equivalence principle (WEP) holds; and (2) that local gravitational and nongravitational experiments are independent of where and when in the universe they are performed. The requirement (9) refers to "when in the universe," since, due to the high degree of homogeneity observed in the universe, we assume that β_a is only time dependent. Furthermore, one experiment is not sufficient to test the SEP; at least two experiments are needed at two different times, because β_a can be normalized to unity at any one time; what is physically relevant is $\dot{\beta}_a$.

3. TENTATIVE EVIDENCE OF A VIOLATION OF SEP

Using 8249 lunar occulatation measurements (Van Flandern, 1981), lunar radar ranging data (Dickey *et al.*, 1980; Calame and Mulholland, 1978), and dynamical determinations of the lunar period (Morrison and Ward, 1975; Goad and Douglas, 1978; and Lambeck, 1977), it has been suggested that at the present epoch $\dot{\beta}_a/\beta_a \simeq 10^{-11}$ year^{-1}. However, as tidal forces make the earth-moon system less than an ideal laboratory for our purposes, it would be more satisfactory to use radar ranging data to the inner planets Mercury, Venus, and Mars. Using the available results, an upper limit $|\dot{\beta}_a/\beta_a| \lesssim 10^{-10}$ year^{-1} was set (Reasenberg and Shapiro, 1978).

We stress here one aspect of the theoretical analysis. It may seem natural to use the standard Einstein equations with the simple alteration $G \to G(t) = G_0 + \dot{G}_0 \Delta t + \cdots$ whenever G appears. However, from the Einstein equations it follows quite generally that for an isolated system, $GM = \text{const}$ (Landau and Lifshitz, 1962; Tolman, 1966), where M is the total mass. [For any local system, this constraint holds regardless of cosmological expansion (Will, 1979).] The violation of this constraint can lead to serious errors. For example, because the period and distance to a planet are given by $P \sim J^3/(GM)^2$, $R \sim J^2/(GM)$, $GM = \text{const}$ implies $\dot{R} = 0$ and $\dot{P} = 0$ and not $2\dot{R}/R = \dot{P}/P = -2\dot{G}/G$, as usually stated. This argument shows that a value of $\dot{G}/G$ cannot be extracted from the standard Einstein equations, where by construction G always appears multiplied by M, the product being required to satisfy the constraint $GM = \text{const}$.

For these reasons, we (in collaboration with R. Helling and P. J. Adams of JPL) have enlarged the system of dynamical equations (originally used by R. Helling) to make them compatible with a possible violation of the SEP. The Viking radar ranging data to Mars are now being analyzed. The best fit solution will hopefully provide a reliable value for $\dot{\beta}_a$.

4. PURPOSE AND BASIC ASSUMPTIONS

Let us construct a theoretical framework that allows for an SEP violation in the form of a nonconstant β_a. This allows for the presence of two fundamental systems of units in nature, which is necessary to explore consistently the implications of an SEP violation as well as to provide relations which can be subjected to observational test.

Gravitational or dynamical units (EU, where E stands for Einstein), are the units in which Einstein equations remain unchanged. These field equations contain the equations of motion that yield the clock equation (7), which is therefore taken to give the gravitational or dynamical unit of time (also called ephemeris time). In EU, $G_{\rm E} = \text{const}$ by definition and so the total mass is also constant, since $G_{\rm E}M_{\rm E}$ is constant. In addition, from $T^{\mu\nu}_{E;\nu} = 0$ applied to a pressureless fluid, we find that the rest mass is also constant. Therefore, in general

$$G_{\rm E} = \text{const}, \qquad M_{\rm E} = \text{const} \tag{10}$$

where M is either total or (macroscopic) rest masses. As the particle number N will be shown not to be constant, equation (10) does not mean that microscopic masses $m_{\rm E}$ are constant in EU. Other atomic quantities, such as e and h, are also in principle not constant in EU.

Atomic units (subscript a) are the units in which the dynamical behavior of an electromagnetic clock is governed by equation (4), which in turn implies that equation (5) is taken to be the atomic unit of time. Furthermore, in AU we have

$$e_{\rm a} = \text{const}, \qquad m_{\rm a} = \text{const}, \qquad h_{\rm a} = \text{const} \tag{11}$$

The first two terms are the analogue of (10) for microphysics. [Actually, terms in equation (11) are contained in equations (4).] In analogy with what we said earlier, the second term of equation (11) does not imply that macroscopic masses M_a are constant in AU. In fact, they are not [see equation (25)]; G_a is also not constant [see equation (18)].

Let us now consider the basic problem of determining the relation of the two "preferred" systems of units. We introduce a language that describes any physical equation in a general system of units of which the two "preferred" systems are special cases.

5. PHYSICS IN GENERAL UNITS

Let us reconsider equation (1) and transform L_* to L_{**},

$$L_* \to L_{**} = \Omega^{-1}(x)L_* = (\Omega_* \Omega)^{-1} L \equiv \Omega_{**}^{-1} L$$

The quantity Ω_* has therefore power -1, as from equation (3), where Λ^*, Λ, and Ω_* are replaced by Ω_{**}, Ω_*, and Ω, respectively. Therefore, any quantity of the form $\Lambda^* \Omega_*^{\pi(\Lambda)}$ has zero power under subsequent scale transformations:

$$\Lambda^* \Omega_*^{\pi(\Lambda)} = \Lambda^{**} \Omega_{**}^{\pi(\Lambda)}$$

As an example, we perform a further transformation of equation (6) to a g_{**}, F_{**}, J_{**} system. The final result can easily be seen to be of the same form as equation (6), with only double starred quantities in it. Therefore, equation (6) can be said to be written in general units.

Let us now define a fiducial system of units by $\Omega_* = 1$, and a general system of units by $\Omega_* = \beta$. Here, as in previous work (Canuto *et al.*, 1977), we choose EU as the fiducial system ($\beta_{EU} = 1$), so that to be consistent with equation (8), the AU system is defined by $\beta_{AU} = \beta_a$. Note that while β, defining a general system of units, has power -1, β_a is of power zero, because $\beta_a = \beta_{AU}/\beta_{EU}$ is the same in all systems of units. Clearly, the use of general units does not introduce any new physics.

6. THE ACTION

To deal with the problem of constructing the two clocks, we propose an action in general units, which as such must be of zero power,

$$I = \int \mu \beta^{2-g} \, ds + \frac{1}{16\pi} \int \beta^{2-g} F_{\alpha\gamma} F^{\alpha\gamma} \sqrt{g} \, dx^4 + \int \beta^{2-g} e A_\nu u^\nu \, ds \tag{12}$$

where μ represents masses in general (microscopic or/and macroscopic) and where the relation between $F_{\mu\nu}\beta^{1-g/2}$ and $A_\alpha \beta^{1-g/2}$ is the usual one.

It is easy to check that the power of I is zero. Because the dimensions of I are $[M][L]$, the β^{2-g} factors are required for $\pi(I)=0$. The matter part of equation (12) is different from that proposed by Dirac (1973), which is a particular case of equation (12) if $\mu\beta^{1-g}=\text{const}$. In AU, and for microscopic masses, this implies $g=1$, because of equation (11). However, $g=1$ will be shown not to allow the two clocks to run at different rates. The relaxing of the restriction $\mu\beta^{1-g}=\text{const}$ is why we can construct two clocks that run at different rates.

7. MACROSCOPIC GRAVITATIONAL CLOCK

Consider the periodic motion of a planet in the gravitational field of a star. From the first term in equation (12), we derive the following equations of motion in general units,

$$u^{\alpha}_{;\gamma}u^{\gamma}+\frac{(\mu\beta^{2-g})_{,\gamma}}{(\mu\beta^{2-g})}\Delta^{\alpha\gamma}=0 \tag{13}$$

where the metric $g_{\mu\nu}$ due to the star is given in general units by the Schwarzschild metric times β^{-2}. As we are dealing with a macroscopic object, then

$$\mu=M, \qquad M_{\mathrm{E}}=\beta^{1-g}M=\text{const} \tag{14}$$

where the second relation is the general law for mass transformation from Einstein units to general units, following from equation (3) with $\Lambda=M_{\mathrm{E}}$, $\Lambda^*=M$, $\Omega_*=\beta$, $\pi(m)=1-g$. The last equality in equation (14) follows from equation (10). Equation (13), with equation (14), specializes to

$$\text{EU:}\quad u^{\alpha}_{;\gamma}u^{\gamma}=0; \qquad \text{AU:}\quad u^{\alpha}_{;\gamma}u^{\gamma}+\frac{\beta_{a,\gamma}}{\beta_a}\Delta^{\alpha\gamma}=0 \tag{15}$$

The solutions for the period P can be easily worked out. The results are

$$\text{EU:}\quad P_{\mathrm{E}}=\frac{2\pi J_{\mathrm{E}}^3}{(G_{\mathrm{E}}M_{\mathrm{E}})^2}=\text{const}$$

$$\text{AU:}\quad P_{\mathrm{a}}=\beta_{\mathrm{a}}^{-1}P_{\mathrm{E}} \tag{16}$$

8. ELECTROMAGNETIC CLOCK

The motion of an electron in the field of a proton $F^{\lambda\nu}$ is governed by the following two equations, derivable from equation (12):

$$(\sqrt{g}F^{\lambda\nu}\beta^{1-g/2})_{,\nu}=4\pi\int e\beta^{1-g/2}\delta^4(x^{\alpha}-z^{\alpha})\,dz^{\lambda}\equiv J^{\lambda} \tag{17a}$$

$$u^{\alpha}_{;\nu}u^{\nu}+\frac{(\mu\beta^{2-g})_{,\nu}}{(\mu\beta^{2-g})}\Delta^{\alpha\nu}=\frac{e}{\mu}u^{\nu}F^{\alpha}_{\nu} \tag{17b}$$

In equation (17b) we used $e\beta^{1-g/2} = \text{const}$, a constraint derivable from equation (17a) using the antisymmetry of $F_{\mu\nu}$. Let us now specify equation (17) to AU and EU. In AU we require equation (17) to coincide with equation (4). In AU, $\beta = \beta_a$, and for a microscopic mass $\mu = m = m_a = \text{const}$. The requirement can therefore be fulfilled only if

$$g = 2, \qquad \pi(m) = -1, \qquad G_a\beta_a^2 = \text{const} \tag{18}$$

To derive the last term of equation (18), we have used equation (3) with $\Lambda \equiv G_E$, $\Lambda^* \equiv G_a$, $\Omega_* \equiv \beta_a$, as well as equation (10).

The corresponding solution for the period p_a in a local Lorentzian coordinate frame is now given by equation (5), with the subscript a attached to all the quantities. Let us now consider EU, where

$$\beta = 1, \qquad \mu \equiv m_E = m_a\beta_a^{\pi(m)}$$
$$e^2 = e_E^2 = e_a^2\beta_a^{1+\pi(m)} \tag{19}$$

where we have again used equation (3). Because $g = 2$, equation (17a) retains the same form as in AU, whereas equation (17b) becomes

$$\text{EU:} \quad u^\alpha_{;\nu}u^\nu - \frac{\beta_{a,\nu}}{\beta_a}\Delta^{\alpha\nu} = \frac{e_a}{m_a}\beta_a u^\nu F^\alpha_\nu \tag{20}$$

Using a local Lorentzian coordinate frame, the solution for the period p_E is found to be

$$p_E = \beta_a p_a \qquad [p_a = \text{const, equation (5)}] \tag{21}$$

To achieve the desired result, we had to a fix a gauge: a relation between β_a and G_a, equation (18). This is a welcome feature because until now, we had to consider g as a free parameter (Canuto and Hsieh, 1979; Canuto *et al.*, 1979). This no longer the case, as the theory now demands equation (18). Note that such a gauge was previously suggested in connection with the 3 K blackbody radiation (Canuto and Hsieh, 1978).

We have proposed a Lagrangian whose solution for the periods of gravitational (P) and atomic (p) clocks are

$$P_E = \beta_a P_a, \qquad p_E = \beta_a p_a, \qquad P_E, p_a = \text{const} \tag{22}$$

or

$$\frac{p_a}{P_a} = \frac{p_E}{P_E} \sim \beta_a \tag{23}$$

namely: in either atomic or gravitational units, the ratio of the periods of the two clocks is not constant, provided β_a is not constant. We have therefore proved that, provided $g=2$, a framework exists which allows the two clocks to run at different rates.

We must stress that the extension of equation (17a) to a charged fluid must be written as $F^{\mu\nu}_{;\nu}=\tilde{J}^{\mu}$, where $\tilde{J}^{\mu}=enu^{\mu}f$, f being an undetermined function of β_a. Due to the antisymmetry of $F^{\mu\nu}$, it follows that $\tilde{J}^{\mu}_{;\mu}=0$, which implies $eNf=\text{const}$. Because for $g=2$, e is constant in any units, equation (19), it then follows that $f\sim N^{-1}$. The Coulomb force now becomes $e^2N^2f^2/r^2$. The analogous gravitational force is GM^2/r^2, with $GM^2=G_EM_E^2=\text{const}$, a result valid for $g=2$. The correspondence between macroscopic Coulomb's and Newton's laws is therefore preserved.

9. WEAK EQUIVALENCE PRINCIPLE

In achieving the result (23), the central part was played by the action (12) and the equations of motion ensuing from it. Since equation (13) is in general units, using $g=2$, the fact that $m\beta^{1-g}=m_E=m_a\beta_a^{1-g}\sim\beta_a^{-1}$, and equation (14), we obtain

$$\text{Microscopic bodies:}\quad u^{\alpha}_{;\nu}u^{\nu}+\frac{\beta_{,\nu}}{\beta}\Delta^{\alpha\nu}=\frac{\beta_{a,\nu}}{\beta_{\alpha}}\Delta^{\alpha\nu} \tag{24a}$$

$$\text{Macroscopic bodies:}\quad u^{\alpha}_{;\nu}u^{\nu}+\frac{\beta_{,\nu}}{\beta}\Delta^{\alpha\nu}=0 \tag{24b}$$

which indicate that the two types of bodies do not follow the same types of trajectories. Equations (24) are still in general units.

From the operational point of view, we are only interested in AU, so we limit our considerations to them. Equations (24) tell us that in AU, microscopic masses follow geodesics while macroscopic masses do not. In either case, however, masses do not enter the equations and the WEP is separately satisfied, in the sense that all macroscopic objects follow the same path as do all microscopic ones. The results of the Eotvos–Dicke–Braginskii experiments (Will, 1979; Rudenko, 1978), showing that two (macroscopic) bars of Al and Au (or Al and Pt) follow the same path, are therefore in full agreement with equations (24), as the extra "force" represented by β_a is independent of the mass and it affects both bars equally. thus, its effect cancels out in this type of experiment.

To test equations (24) one should follow in time the trajectories of an atom and of a planet, which is the procedure in the radar ranging experiments. In fact, one may think of atomic and gravitational clocks (the earth–moon system) as two "objects" moving in space-time following two given trajectories. If, as time evolves, the two systems follow different types of trajectories, charting the time evolution of the macroscopic object with the reference provided by the microscopic object cannot yield constant results if the ratio of the proper lengths spanned by the two objects is not constant in time. Therefore equations (24) are an alternative way of interpreting the lunar and planetary data that stresses the difference in the two trajectories rather than the difference of the two clocks. These two ways of interpreting the data are equivalent, although the second one is the one almost exclusively referred to in this context.

10. PARTICLE NUMBER NONCONSERVATION

The fact that only $g = 2$ is allowed has important consequences. In fact, since m_a and M_E are constant, it follows that [using equation (3) between AU and EU]

$$m_E \sim \beta_a^{1-g}, \qquad M_a \sim \beta_a^{g-1}, \qquad N \sim \beta_a^{g-1} \tag{25}$$

implying that N is no longer constant. To have a conserved N, we have to choose either $\beta_a = \text{const}$, in which case the SEP is automatically satisfied, or $g = 1$, which would also lead us to an SEP-conserving framework. In fact, for $g = 1$ the left-hand side of equation (15) governing the macroscopic gravitational clock in AU would be identical to that of equation (17b) governing a microscopic electromagnetic clock in AU, thus leading to no difference between the two periods P_a and p_a, thus returning to the SEP. [The right-hand side of (17b) does not affect this statement because, due to spherical symmetry, it does not affect the angular momentum conservation law.]

Equation (25) is the most important consequence of the SEP violation framework, as it indicates that a violation of the SEP is intimately related to a violation of the particle number conservation law.

11. GRAVITATIONAL CONSTANT

It is often stated that if atomic and gravitational clocks are different, the gravitational constant G must vary with atomic time. While the statement is not incorrect, it may give the impression that it adds some new fact. This is not the case. In fact, neither in the gravitational action of Canuto *et al.* (1977) nor in the one presented here is there an independent function G. One calls G the combination $G_E\beta^{-g} = G$, $G_E = \text{const}$. But one does not introduce new physics, one simply lumps together a function β, a parameter

g, and a constant G_E. That the physics is contained in β_a and not in G_a is evident from the fact that the experiments on the moon and the inner planets yield directly $\dot{\beta}_a$ and not $\dot{G}_a$, which is derived quantity, equation (18).

12. COSMOLOGICAL MEANING OF β_a

The framework presented here, while permitting the examination of the implications of an SEP violation, does not explain the physical mechanism behind it. In fact, β_a is treated here as an external quantity, much as viscosity is treated in classical fluid mechanics, where it enters as an external parameter whose evaluation requires a microscopic kinetic theory.

Although we do not offer a dynamics for β_a, it is important to stress that a dynamics of β_a can be either of a local or global nature. In the first case, β_a is regarded as a space-time field described by an action to be added to the total action; this would entail a coupling of β_a to local matter, with the result that even in EU, macroscopic gravity would no longer be described by standard Einstein equations, thus departing completely from our basic assumptions. A local approach was adopted by Brans and Dicke (BD). As several studies have indicated (Weinberg, 1972, pp. 628–629; Bekenstein, 1989), solar system experiments constrain, within the BD approach, the variability of β_a to some orders of magnitude below the value quoted previously. As there is no reason why we should arbitrarily restrict $\dot{\beta}_a$ to such low values, we find the local approach inadequate.

The value of $\dot{\beta}_a$ implying $\dot{\beta}_a/\beta_a \sim H_0$ suggests that β_a is related to the structure of the universe and that its dynamics is likely to be governed by topological rather than local space-time considerations. The SEP violation represents therefore a cosmological influence on the local physics, in accord with Mach's principle. In contrast, accepting the SEP as an exact law of physics is equivalent to assuming that local physics is independent of the rest of the universe.

Recent work on nucleosynthesis (Canuto and Goldman, 1982) has indicated that, as expected, an SEP violation with a time scale of the order of the Hubble time cannot be extrapolated back to the radiation-dominated era. Furthermore, a dynamics for photons can be constructed (Canuto and Goldman, 1982) independently of β_a: in particular, a very general argument has been found indicating that the photon number N_γ, contrary to the particle number N, equation (25), is adiabatically conserved for any value of the parameter g. [The photon treatment presented in Canuto and Hsieh (1979) is therefore valid only if $g = 1$.]

The two previous results suggest that an SEP violation, if it exists, began to manifest itself only after the universe entered the matter-dominated era, before which β_a may have been constant.

13. CONCLUSIONS

The most important results of the present analysis are:

- Einstein field equations retain their standard form only in EU. In AU, they depend on β_a and their form is given by equation (2.34) of Canuto *et al.* (1977).
- The trajectory of a macroscopic (many-body) object is a geodesic in EU; in AU, β_a factors enter. The results are given in equation (24b).
- The trajectory of a microscopic (one-body) object is a geodesic in AU; in EU, β_a factors enter [equation (24a)].
- While in AU the description of a microscopic one-body dynamics is unchanged (this holds true even at the level of the Schrödinger equation), the description of a many-body situation is affected by β_a. In fact, the particle number $N \sim \beta_a$, equation (25). This in turn implies that macroscopic masses are such that $M_a \sim \beta_a$, $M_E \sim$ const. Microscopic masses are such that $m_a \sim$ const, $m_E \sim \beta_a^{-1}$. Finally, the gravitational coupling G is such that $G_E =$ const, $G_a \sim \beta_a^{-2}$.

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